Honors Distinction Project Presents

MATHEMATICAL COMPOSITIONS AND MUSICAL CALCULATIONS

A Project Created by Liv Long

INTRODUCTION

György Ligeti wrote a piece entitled Désordre, which is a piece of music designed to demonstrate Chaos Theory. Throughout the piece, notes are added and dropped at intervals which appear random. Since Chaos Theory is ordered disorder, this process can be easily replicated with the analysis and use of famous musical literature. I have expanded this idea to also use fractals, Fourier series, and other techniques to keep demonstrating mathematics, since the average person understands and enjoys music more then they either understand or enjoy math. I have taken these higher level mathematics and then taken them farther to look at them through the lens of music and musical composition.

Please enjoy!

WELCOME

Welcome
by Liv Long on Tenor Saxophone:
Welcome! is a piece that displays graphs of basic functions such as x, x^2, x^3, x^4, sinx, and cosx. Each phrase of the song is a musical representation of the graphical curves with each slightly different from the last.
This is just an introduction to the wonderful world of mathematical compositions.

FRACTALS

Fractal Introduction by Eric Lee on Clarinet:
Fractals are self-similar objects that as they grow they incorporate the same shape into itself. This piece is a basic musical fractal. At each iteration the longest notes in the pattern are replaced by a smaller version of the same pattern.

Koch’s Snowflake by Liv Long on Flute:
The piece is a representation of Koch’s Snowflake up to the third iteration. Koch's Snowflake is a triangular fractal that increases by an additional triangle in the middle of each line for each new iteration. Between iterations there is three beats of rest so that the listener can understand how much more complicated the piece becomes as the fractal progresses.

Sierpinski Set Free by Liv Long on Flute:
Sierpinski's triangle is a fractal that takes a triangle and draws lines between the midpoints of each side of the triangle. The music travels along the bottom of the triangle followed by the left side and then the right. For each iteration a new point is added exactly half-way between the existing points.

CHAOS THEORY

Chaotic Bassoons
by Liv Long on Bassoon:

Chaotic equations begin by choosing a value and each new value builds off the previous one. The music shows three examples of possible situations with different starting values: These initial conditions differ by 0.001 and yet they take completely different tracks. The music shows these situations by each number being associated with a note on the staff. The music plays each version individually and then at the same time to help give perspective on the differences between the situations.

Mary had a Crazy Lamb
by Liv Long on Tenor Saxophone:

The chaotic equation $x(n+1)=4\,[x(n)][1-x(n)]$ produces values that can then be applied to a song such as “Mary had a Little Lamb”. For each of these values that were found, the selected note was raised by one pitch. The original song will be played first followed by the chaotically adjusted version.

FOURIER SERIES

Waving, Square Waving
by Liv Long on Bassoon:
The music piece is based around the square curve and how the Fourier Series forms around that. Fourier Series is a mathematical function that attempts to recreate a specific curve. The first two notes that you hear are the square curve. Following a measure of rest is the first iteration of Fourier Series, then the second iteration, and so on. It is easy to hear how much closer the estimations grow toward the square curve.

FIBONACCI SEQUENCE

Fibonacci's Rhythm
by Liv Long on Tenor Saxophone:
Fibonacci's Sequence is based on the principle that each new element in a sequence is the sum of the previous two elements. To apply this to music, each new measure will be a combination of the two immediately before it. There are two examples of this within the piece separated by a measure of rest.

MODULAR ARITHMETIC

Modularity
by Liv Long on Bassoon:
Modularity is a piece that takes a basic scale and arpeggio and shows how the different modulos affect the range of the piece. Modulos are created by changing the number of possible values. The first time through the scale is played regularly, then with eight different notes, then six, then four, and finally two. This shows how different notes become equivalent.

Modular the Beautiful
by Eric Lee on Clarinet:
The same concept is used except that the piece is more recognizable, and the modulos is more complicated. The piece is played as written the first time through, then with eight notes and again with four.

SYMMETRIES

Musical Symmetries
by Luke Elder on Piano:
The piece is created using this concept by having four notes written on a square and then the same symmetries applied. Squares have eight symmetries including turns of 90, 180, 270, and 360 degrees, horizontal flips, vertical flips, and two diagonal flips. The music reads for each measure to be a "square" and then the chart is read left to right. The result is intriguing to hear and to understand that it is a collection of eight different measures strategically placed and heard to understand how the square is also changed and moved.

Triangularity
by Liv Long on Bassoon:
The second piece is created in a very similar manner, only a triangle was used. Triangles have different symmetries than squares and the piece reflects this difference. The square reflections result in very different measures and notes while the triangle symmetries all stay within a certain range.

CLOSING

Mathematical Mountains
by Liv Long on Tenor Saxophone, Flute, and Bassoon:
This piece combines all of the aspects of mathematics that have been explored in this project. A fractal has been created out of a square wave where every eighth note following the first has been expanded to multiple notes. The number of notes is determined by Fibonacci's sequence. When the second person begins playing, they are playing the same phrases but in modulo 4. And the third person who comes in is playing the phrase again, but with chaotic notes raised. The piece ends with the musicians playing in sync of a vertical flip of the beginning of the phrase.

CONCLUSION

This has been such a rewarding project and I have loved getting to work with the mathematics, music, and musicians. Thank you for listening.

I would like to also thank Dr. Minton, Dr. LaChance, and Dr. Poli of Roanoke College for serving on my committee as well as the Roanoke College Honors Program for providing me with this opportunity. Thank you to all the musicians who have played my pieces. And thank you to my family and friends who have supported me, given me advice, and encouraged me as I faced struggles throughout this process.

This project is being turned into a website so that students of all ages can learn to love math and music as much as I do. I hope that this project can convince at least one person, that math is cool.

https://mathmusic.pages.roanoke.edu/

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Thursday the 24th of September
5:30PM
Olin Amphitheater